***MATRIX TRANSFORMATIONS***

***Summary:***

***1.*** *A transformation describes the relation between any point and its image point*

***2.*** *In transformation****,*** *the original shape is called the object and the transformed shape is called the image*

***3.*** *Transformation × object = image* ***(T × O = I)***

***4.*** *From* ***T × O = I,*** *it follows that  Thus the inverse of transformation maps the image back to the object****.***

***5.*** *Image area = object area × det of transformation. Thus the ratio of ratio of object area to image area =* ***1: det of transformation***

***6.*** *Two successive transformations*  *followed by*  *can be written as a single matrix*  *The multiplication is in reverse order because* *first maps the object onto an intermediate image then*  *gives the final image*

***7.*** *A unit square is one with vertices at points* ***O(0, 0), I(1, 0), J(0, 1)*** *and* ***K(1, 1)***

*as shown****:***

***0***

***1***

***1***

***I***

***J***

***K***

*The images of* ***I(1, 0)*** *and* ***J(0, 1)*** *can be used to obtain certain types of transformation matrices****.***

***EXAMPLES:***

***1.*** *A transformation matrix*  *maps point* ***P(−1, 2)*** *onto its image* ***.*** *Find the****:***

***(i)*** *coordinates of* 

***(ii)*** *transformation matrix which maps* *onto* ***P***

***2.*** *A transformation matrix* *maps point* ***P(−1, 3)*** *on to its image*

*Find the values of* ***b*** *and* ***c.***

***3.*** *A transformation matrix*  *maps point* ***Q*** *onto*  *Find the*

*coordinates of* ***Q***

***4.*** *A Triangle with vertices* ***A(2, 3), B(4, 5)*** *and* ***C(6, 3)*** *is mapped onto triangle*

 *by a transformation represented by matrix*  *Find the****:***

***(i)*** *coordinates of*  *and* 

***(ii)*** *area of triangle* ***ABC.*** *Hence find the area of triangle* 

***(iii)*** *transformation matrix which maps*  *back to* ***ABC***

***5. (i)*** *Find the image of a unit square under the transformation matrix .*

***(ii)*** *plot the unit square and its image in* ***(i)*** *above on the same axes.*

***6.*** *A transformation represented by matrix*  *maps triangle* ***ABC*** *onto its image with vertices*  *and*  *Find the****:***

***(i)*** *coordinates of* ***A, B*** *and* ***C***

***(ii)*** *area of triangle* ***ABC.*** *Hence find the area of triangle* 

***7.*** *A triangle with vertices* ***A(−4, −2), B(−2, −2)*** *and* ***C(−2, −4)*** *is mapped onto its image by a transformation to give*   *and*  *Find the****:***

***(i)*** *matrix for the transformation****.***

***(ii)*** *ratio of the area of triangle* ***ABC*** *to that of triangle* 

***(iii)*** *transformation matrix which maps*  *back to* ***ABC***

***8****. An object of areas is transformed by the matrix  Find the area of its image*

***9.*** *A transformation matrix  maps an object of areas  onto an*

*image of area  find the values of* ***x.***

***10.*** *A transformation matrix  maps the line* ***y = 3x − 5****onto line* ***L.*** *Find the equation of the line* ***L.***

***11.*** *Two transformations are represented by the matrices  and  Find the matrix of a single transformation which represents****:***

***(i)*** *followed by* 

***(ii)*** *followed by* 

***12.*** *A Triangle with vertices* ***A(2, 3), B(4, 5)*** *and* ***C(6, 3)*** *is mapped onto triangle*

 *after two successive matrix transformations  followed by  Find the****:***

***(i)*** *matrix of a single transformation which maps* ***ABC*** *onto* 

***(ii)*** *coordinates of*  *and* 

***(ii)*** *area of triangle* ***ABC.*** *Hence find the area of triangle* 

***(iii)*** *matrix transformation that will map*  *back to* ***ABC***

***13.*** *A triangle with vertices* ***P(0, 2), Q(1, 4)*** *and* ***R(2, 2)*** *is mapped on its image* *by the matrix transformation *  *Triangle* *is then mapped onto* *by another matrix transformation *  *Find the****:***

***(i)*** *coordinates of* *and* 

***(ii)*** *matrix transformation that will map*  *back to* ***PQR***

***(iii)*** *ratio of the area of triangle* ***PQR*** *to that of triangle* 

***EXERCISE:***

***1.*** *A transformation matrix* *maps point* ***P(3, −2)*** *on to its image* *Find the values of* ***b*** *and* ***c.***

***2.*** *A transformation matrix*  *maps a triangle with vertices* ***P(3. 1), Q(5, 5)*** *and* ***R(7, 1)*** *onto its image.*  *Find the area of the image triangle*

***3.*** *A transformation matrix  maps an object of areas  onto an*

*image of area  find the values of* ***x.***

***4.*** *A transformation matrix  maps the line* ***2y = x − 1*** *onto line* ***L.*** *Find the equation of the line* ***L.***

***5.*** *Find the image of a unit square under the transformation matrix *

***6.*** *Find the image of point* ***P(5, 3)*** *after a transformation matrix*  *followed by* 

***7.*** *A transformation matrix maps point* ***(1, 2)*** *onto**and* ***(2, 3)*** *onto* *Find the****:***

***(i)*** *matrix for the transformation****.***

***(ii)*** *image of point* ***P(−3, −2)*** *under the transformation above*

***8.*** *A triangle with vertices* ***P(2, 1)******Q(2, 3)*** *and* ***R(4, 1)*** *is mapped on its image* *by a translation *  *Triangle* *is then mapped onto* *by a matrix transformation *

***(i)*** *Find the**coordinates of* *and* 

***(ii)*** *Plot triangle* ***PQR*** *and its image* *on the same axes****.***

***(iii)*** *By joining triangle* ***PQR*** *and its image* *find the volume of the resulting figure formed*

***9.*** *A transformation represented by matrix*  *maps triangle* ***ABC*** *onto its image with vertices*  *and*  *Find the****:***

***(i)*** *coordinates of* ***A, B*** *and* ***C***

***(ii)*** *area of triangle* ***ABC.*** *Hence find the area of triangle* 

***10.*** *A transformation represented by matrix*  *maps triangle* ***ABC*** *onto its image with vertices*  *and*  *Triangle* *is then mapped onto* *by another matrix transformation  Find the****:***

***(i)*** *coordinates of* ***ABC*** *and* 

***(ii)*** *matrix of a single transformation which maps*  *back to* ***ABC***

***11.*** *A Triangle with vertices* ***(−2, 1), Q(3, 1)*** *and* ***R(0, 3)*** *is mapped onto its image triangle by the matrix transformations  followed by  Find the****:***

***(i)*** *coordinates of the final image*

***(ii)*** *matrix transformation that will map the final image back to the object*

***(i)*** *area of the final image*

***12.*** *A triangle with vertices* ***P(2,0), Q(1, −3)*** *and* ***R(−2, 1)*** *is mapped on its image* *by the matrix transformation *  *Triangle* *is then mapped onto* *by another matrix transformation *  *Find the****:***

***(i)*** *coordinates of* *and* 

***(ii)*** *matrix transformation that maps* ***PQR***  *onto* 

***(iii)*** *ratio of the area of triangle* ***PQR*** *to that of triangle* 

***14.*** *A Triangle with vertices* ***A(1, 3), B(2, −1)*** *and* ***C(4, 0)*** *is mapped onto its image by a matrix transformation to give*   *and*   *Find the****:***

***(i)*** *transformation matrix*

***(ii)*** *area of triangle* ***ABC.*** *Hence find the area of triangle* 

***(iii)*** *transformation matrix that will map*  *back to* ***ABC***

***REFLECTION***

***Summary:***

***1.*** *In reflection****:***

***(i)*** *the image is formed using a mirror line*

***(ii)*** *the image is as far behind the mirror as the object is in front of it*

***(iii)*** *one figure is the mirror image of the other*

***2.******(i)*** *Reflection is described by stating the mirror line*

***(ii)*** *The most common reflections can be described by a* ***2 × 2*** *matrix using the images of* ***I(1, 0)*** *and* ***J(0, 1)*** *of a unit square*

***EXAMPLES:***

***1.*** *Use the points* ***I(1, 0)*** *and* ***J(0, 1)*** *to find the matrix corresponding to****:***

***(i)*** *a reflection in the line* ***y = 0 ( x−axis)***

***(ii)*** *a reflection in the line* ***x = 0 ( y−axis)***

***(iii)*** *a reflection in the line* ***x + y = 0 (y = −x )***

***(iv)*** *a reflection in the line* ***x − y = 0 (y = x )***

***2.*** *A triangle with vertices* ***P(1, 4), Q(3, 2)*** *and* ***R(5, 3)*** *is mapped onto its image by a reflection in the line* ***y = 0***

***(a)*** *Write down the matrix for the reflection*

***(b)*** *Find the coordinates of the image of* ***PQR:***

***(i)*** *using matrices*

***(ii)*** *by construction*

***3.*** *A triangle with vertices* ***P(2, 3), Q(5, 4)*** *and* ***R(5, 6)*** *is mapped onto its image by a reflection in the line* ***x = 0***

***(a)*** *Write down the matrix for the reflection*

***(b)*** *Find the coordinates of the image of* ***PQR:***

***(i)*** *using matrices*

***(ii)*** *by construction*

***4.*** *A triangle with vertices* ***P(2, −4), Q(6, −3)*** *and* ***R(3, −1)*** *is mapped onto its image by a reflection in the line* ***x − y = 0***

***(a)*** *Write down the matrix for the reflection*

***(b)*** *Find the coordinates of the image of* ***PQR:***

***(i)*** *using matrices*

***(ii)*** *by construction*

***5.*** *A triangle with vertices* ***P(2, 3), Q(5, 4)*** *and* ***R(5, 6)*** *is mapped onto its image by a reflection in the line* ***x + y = 0***

***(a)*** *Write down the matrix for the reflection*

***(b)*** *Find the coordinates of the image of* ***PQR:***

***(i)*** *using matrices*

***(ii)*** *by construction*

***6.*** *Find the coordinates of the image of point* ***P(3, 2)***  *under a transformation*  *followed by a reflection in a line* ***y = −x***

***7.*** *Find the coordinates of the image of a triangle with vertices* ***A(2, 1)******B(2, 3)*** *and* ***C(4, 1)*** *under a translation  followed by a reflection in the* ***x−*** *axis*

***8.(a) (i)*** *Find the coordinates of the image of a triangle with vertices* ***A(1, 4)******B(1, 1)*** *and* ***C(2, 1)*** *under a transformation matrix *

***(ii)*** *Draw triangle* ***ABC*** *and its image* *on the same axes*

***(iii)*** *Describe the matrix transformation* ***L.*** *Hence deduce the matrix transformation which would map triangle*  *onto triangle* ***ABC.***

***(b)*** *Triangle*  *is mapped onto triangle* *by a matrix*

*transformation *

***(i)*** *Find the coordinates of* 

***(ii)*** *Draw triangle* *on the same axes in* ***a(ii)*** *above.*

***(iii)*** *Use your graph to describe a single transformation that will map triangle*

***ABC*** *onto triangle*  *Hence find the single matrix transformation which maps triangle* ***ABC*** *onto triangle* 

***9.*** *Find the equation of the image of the line* ***y = 2x −1*** *when reflected in the line* ***x + y = 0***

***10.*** *A triangle with vertices* ***P(3, 2), Q(1, 4)*** *and* ***R(5, 3)*** *is mapped onto its image*   *and*  *by a transformation* ***T.*** *Triangle* *is then mapped onto triangle*  *by another matrix transformation *

***(i)*** *Draw triangle* ***PQR*** *and its image* *on the same axes*

***(ii)*** *Describe fully the transformation which maps* ***PQR*** *onto****.*** *Hence write down the matrix for the transformation*

***(iii)*** *Find the coordinates of* 

***(iv)*** *Find the single matrix of transformation which maps* ***PQR*** *onto* 

***EER:***

***1.*** *The image of* ***P(3, 2)*** *after a reflection is*  *Use a graph paper to**construct the line of reflection and state its equation*

***2.*** *A triangle with vertices* ***P(2, −4), Q(6, −3)*** *and* ***R(3, −1)*** *is reflected in the line* ***x − y = 0*** *to get triangle*  *Triangle* *is then reflected in the line* ***x = 0*** *to get triangle* 

***(i)*** *Draw the three triangles on the same axes****.******[****Use a scale of* ***2cm*** *to* ***1*** *unit****]***

***(ii)*** *Write down the coordinates of* *and* 

***3.*** *A triangle with vertices* ***P(3, 2), Q(1, 4)*** *and* ***R(5, 3)*** *is mapped onto its image*   *and*  *by a transformation* ***.*** *Triangle* *is then mapped onto triangle*  *by another matrix transformation *

***(i)*** *Draw triangle* ***PQR*** *and its image* *on the same axes*

***(ii)*** *Describe fully the transformation which maps* ***PQR*** *onto****.*** *Hence write down the matrix for the transformation*

***(iii)*** *Find the coordinates of* 

***(iv)*** *Find the single matrix of transformation which maps* ***PQR*** *onto* 

***ENLARGEMENT***

***Summary:***

***1. (i)*** *Enlargement is a transformation which changes the size of an object*

***(ii)*** *Enlargement is described by stating its centre and the scale factor*

***2.*** *The point about which enlargement occurs is called the centre of enlargement*

***3.******(i)*** *By definition****,*** *scale factor* 

***(ii)*** *The scale factor tells us by how much the object has been enlarged****.*** *Thus a scale factor of* ***3*** *means that the image is three times the size of the object*

***(iii)*** *The sketch below shows how to locate the image* *of point* ***P*** *after an enlargement with centre* ***O*** *and scale factor* ***3:***

***O***

***P***

******

*In the above sketch****,*** *distance* 

***(iv)*** *A positive scale factor means that both the object and its image are on the same side of the centre*

***(v)*** *A negative scale factor means that the object and its image are on the opposite sides of the centre*

***4.*** *Under enlargement****,*** *Image area = Object area ×* 

***EXAMPLES:***

***1.*** *A triangle with vertices* ***P(−1, −2), Q(−3, −4)*** *and* ***R(−5, −1)*** *is mapped onto its image*  *by an enlargement with centre* ***C(−6, −3)*** *and scale factor* ***3.*** *Find the****:***

***(i)*** *coordinates of* 

***(ii)*** *area of triangle* ***PQR.***  *Hence find the area of triangle* 

***2.*** *A quadrilateral with vertices* ***P(3, −9), Q(5, −7), R(3, −6)*** *and* ***S(1, −6)*** *is mapped onto its image*  *by an enlargement with centre* ***C(2, −4)*** *and scale factor* ***−3.*** *Find the****:***

***(i)*** *coordinates of* 

***(ii)*** *area of quadrilateral* ***PQRS .*** *Hence obtain the area of* 

***3.*** *A triangle with vertices* ***P(−6, −2), Q(−2, −2)*** *and* ***R(−2, −6)*** *is mapped onto its image*  *by an enlargement with centre* ***C(0, 4)*** *and scale factor* ***0⋅5.*** *Find the****:***

***(i)*** *coordinates of* 

***(ii)*** *area of triangle* ***PQR.*** *Hence obtain the area of triangle* 

***4.*** *A triangle with vertices* ***P(−6, −2), Q(−2, −2)*** *and* ***R(−2, −6)*** *is mapped onto its image*  *by an enlargement with centre* ***C(0, 2)*** *and scale factor* ***1⋅5.*** *Find the****:***

***(i)*** *coordinates of* 

***(ii)*** *area of triangle* ***PQR .*** *Hence obtain the area of triangle* 

***ENLARGEMENT ABOUT THE ORIGIN***

***Summary:***

*If the centre of enlargement is located at the origin****,*** *then the general matrix for the enlargement is  where* ***k*** *is the scale factor of enlargement*

***EXAMPLES:***

***1.*** *A triangle with vertices* ***P(1, 1), Q(3, 1)*** *and* ***R(1, 4)*** *is mapped onto its image* *by an enlargement with centre* ***O(0, 0)*** *and scale factor* ***3.***

***(a)*** *Write down the matrix for the enlargement*

***(b)*** *Find the****:***

***(i)*** *coordinates of* 

***(ii)*** *area of triangle* ***PQR .*** *Hence obtain the area of triangle* 

***2.*** *A triangle with vertices* ***P(2, 0), Q(4, 1)*** *and* ***R(2, 2)*** *is mapped onto its image* *by an enlargement with centre* ***O(0, 0)*** *and scale factor* ***−2.***

***(a)*** *Write down the matrix for the enlargement*

***(b)*** *Find the****:***

***(i)*** *coordinates of* 

***(ii)*** *area of triangle* ***PQR .*** *Hence obtain the area of triangle* 

***3.*** *A triangle with vertices* ***A(1, 0), B(0, 1)*** *and* ***R(2, 1)*** *is mapped onto its image by a transformation to give*   *and* 

***(i)*** *Find the matrix for the transformation****.***

***(ii)*** *Describe the matrix for the transformation*

***FINDING THE CENTRE AND SCALE FACTOR OF ENLARGEMENT***

***Summary:***

***1.*** *When the object and its image are given****,*** *the centre of enlargement is located as follows****:***

***(i)*** *Join up a point and its image*

***(ii)*** *Repeat for another point and its image*

***(iv)****These lines meet at the centre of enlargement*

***2.*** *To find the scale factor****,*** *use scale factor* 

***EXAMPLES:***

***1.*** *Find the scale factor and centre of enlargement that maps a triangle with vertices* ***P(3, 3), Q(5, 2)*** *and* ***R(5, 4)*** *onto its image*   *and* 

***2.*** *Find the scale factor and centre of enlargement that maps a line with end points* ***P(3, −9)*** *and* ***Q(5, −7)*** *onto its image*   *and* 

***3.*** *Under an enlargement of scale factor* ***3,*** *the image of point* ***P(3, 4)*** *is*  *Find the coordinates of the center of enlargement*

***Soln:***

*Let* ***(x, y)*** *be the required centre*

*If scale factor* 

***⇒****also* 

*∴ x = 2, y = 1*

*Required centre =* ***(2, 1)***

***4.*** *Under an enlargement of scale factor* ***−2,*** *the image of point* ***P(4, −7)*** *is*  *Find the coordinates of the centre of enlargement*

***Soln:***

*Let* ***(x, y)*** *be the required centre*

***⇒****also* 

*∴ x = 3, y = −4*

*Required centre =* ***(3, −4)***

***5.*** *Under an enlargement of scale factor* ***−2,*** *point* ***P(4, 5)*** *is mapped on to* *and* ***Q(3, 6)*** *is mapped on to*  *Find the coordinates of****:***

***(i)*** *the centre of enlargement*

***(ii)*** 

***Soln:***

***(i)*** *Let* ***(x, y)*** *be the required centre*

***⇒****also* 

*∴ x = 2, y = 2*

*Required centre =* ***(2, 2)***

***(ii)*** *By graphical method****,*** 

***EER:***

***1.*** *Under an enlargement of scale factor* ***3,*** *the image of point* ***P(−2, 3)*** *is*  *Find the coordinates of the center of enlargement.*

***2.*** *A triangle with vertices* ***P(2, 4), Q(4, 3)*** *and* ***R(4, 5)*** *is mapped onto its image*  *by an enlargement with centre* ***C(2, 2)*** *and scale factor* ***−2.*** *Find the****:***

***(i)*** *coordinates of* 

***(ii)*** *area of triangle* ***PQR.*** *Hence obtain the area of triangle* 

***3.*** *Point* ***Q(1, 3)*** *is mapped**onto its image* *by an enlargement with centre* ***O(0, 0)*** *and scale factor* ***2.***

***(i)*** *Write down the matrix of enlargement.*

***(ii)*** *find the coordinates of* 

***4.*** *A triangle with vertices* ***P(−2, 1), Q(3, 1)*** *and* ***R(0, 3)*** *is mapped on its image* *by the matrix transformation *  *Triangle* *is then mapped onto* *by another matrix transformation *

***(i)*** *Find the coordinates of* *and* 

***(ii)*** *Describe fully the single**matrix transformation that* ***PQR*** *onto* 

***(iii)*** *Find the area of triangle* 

***5.*** *A triangle with vertices* ***P(1, 1), Q(3, 2)*** *and* ***R(2, 4)*** *is mapped onto its image*  *by an enlargement with centre* ***C(−1, −1)*** *and scale factor* ***2.*** *Find the****:***

***(i)*** *coordinates of* 

***(ii)*** *area of triangle* ***PQR.*** *Hence obtain the area of triangle* 

***6.*** *A triangle with vertices* ***P(0, 2), Q(1, 4)*** *and* ***R(2, 2)*** *is reflected in the line* ***x − y = 0*** *to give triangle*  *Triangle* *is then mapped onto* *by an enlargement with centre* ***O(0, 0)*** *and scale factor* ***−2.***

***(a)*** *Write down the matrix for the****:***

***(i)*** *reflection* ***(ii)*** *enlargement*

***(b)*** *Find the coordinates of* *and* 

***(c)*** *Find the matrix of a single transformation which would map* ***PQR*** *onto* 

***7.*** *Find the scale factor and centre of enlargement that maps a triangle with vertices* ***P(1, 2), Q(3, 3)*** *and* ***R(0, 3)*** *onto its images*  *and* 

***8.*** *Under an enlargement of scale factor* ***3,*** *the image of point* ***P(−1, 2)*** *is*  *Find the coordinates of the center of enlargement.*

***9.*** *A triangle with vertices* ***P(3, 3), Q(2, 5)*** *and* ***R(2, 2)*** *is mapped onto its image*   *and*  *by an enlargement****.*** *Find the scale factor and centre of enlargement*

***10.*** *Under an enlargement of scale factor* ***3,*** *the image of point* ***P(0, 3)*** *is* *Find the coordinates of the center of enlargement.*

***11.*** *Find the scale factor and centre of enlargement that maps a line with end points* ***P(2, 0)*** *and* ***Q(3, 1)*** *onto its image*   *and* 

***12.*** *A quadrilateral with vertices* ***P(3, −9), Q(5, −7), R(3, −6)*** *and* ***S(1, −6)*** *is mapped onto its image*   *and*  *by an enlargement****.*** *Find the****:***

***(i)*** *scale factor and centre of enlargement*

***(ii)*** *area of* ***PQRS .*** *Hence obtain the area of* 

***ROTATION***

***Summary:***

***1.*** *In rotation****,*** *an object is turned about a point called the centre of rotation*

***2.******(i)*** *Rotation is described by stating its centre****,*** *angle of rotation and direction of rotation****.***

***3.******(i)*** *Anticlockwise rotation is positive and clockwise rotation is negative*

***(ii)*** *A* ***90°*** *anticlockwise rotation is called a positive quarter turn*

***(iii)*** *A* ***90°*** *clockwise rotation is called a negative quarter turn*

***(iv)*** *A* ***180°*** *rotation is called a half turn*

***(v)*** *A rotation of* ***360°******(****full rotation****)*** *takes the object back to its original position*

***(vi)*** *A rotation of* ***270°*** *in any given direction is the same as a rotation of* ***90°*** *in the opposite direction*

***4.*** *The sketch below shows how to locate the image* *of point* ***P*** *after an anticlockwise rotation through an angle* ***θ***  *about point* ***O:***

***θ***

***O***

***P***

******

*In the above sketch****,*** *distance* *and* 

***EXAMPLES:***

***1.*** *A line segment with end points* ***P(4, 5)*** *and* ***Q(0, 7)*** *is mapped onto its image by a positive quarter turn about point* ***(2, 1).*** *Find the coordinates of the image of* ***PQ***

***2.*** *Find the coordinates of the image of a line segment with end points* ***P(−2, 4)*** *and* ***Q(1, 7)*** *after a negative quarter turn about point* ***(3, 2)***

***3.*** *Find the coordinates of the image of a line segment with end points* ***P(5, −6)*** *and* ***Q(4, −1)*** *after a half turn about point* ***(1, −2)***

***4.*** *Find the coordinates of the image of point* ***P(5, 4)*** *after a clockwise rotation of* ***270 °*** *about point* ***(3, 2)***

***5.*** *Find the coordinates of the image of a triangle with vertices* ***P(5, 4), Q(8, 4)*** *and* ***R(5, 7)*** *after a positive quarter turn about point* ***(3, 2)***

***ROTATION ABOUT THE ORIGIN***

***Summary:***

*If the centre of rotation is located at the origin****,*** *then this rotation can be described by a* ***2 × 2*** *matrix using the images of* ***I(1, 0)*** *and* ***J(0, 1)*** *of a unit square*

***EXAMPLES:***

***1.*** *Use the points* ***I(1, 0)*** *and* ***J(0, 1)*** *to find the matrix corresponding to****:***

***(i)*** *a positive quarter turn about the origin*

***(ii)*** *a negative quarter turn about the origin*

***(iii)*** *a half turn about the origin*

***2.*** *Given that* ***H*** *denotes a half turn about the origin and* ***X*** *denotes a reflection in the x−axis****,*** *find a single matrix transformation equivalent to* ***XH***

***3.*** *A triangle with vertices* ***P(3, 2), Q(8, 4)*** *and* ***R(5, 7)*** *is mapped onto its image by a positive quarter turn about the origin****.***

***(a)*** *Write down the matrix for the rotation*

***(b)*** *Find the coordinates of the image of* ***PQR***

***4.*** *Find the coordinates of the image of a triangle with vertices* ***P(3, 2), Q(6, 2)*** *and* ***R(6, 5)*** *after a negative quarter turn about the origin*

***5.*** *Find the coordinates of the image of a triangle with vertices* ***P(8, 6), Q(4, 10)*** *and* ***R(2, 6)*** *after a half turn about the origin*

***6.*** *A triangle with vertices* ***P(2, 1), Q(4, 4)*** *and* ***R(2, 4)*** *is reflected in the line* ***y = 0*** *to get triangle*  *Triangle* *is then given a negative quarter turn about the origin to get triangle* 

***(i)*** *Draw the three triangles on the same axes****.******[****Use a scale of* ***2cm*** *to* ***1*** *unit****]***

***(ii)*** *Write down the coordinates of* *and* 

***(iii)*** *Use your graph to describe fully the transformation which maps* *back onto* ***PQR.***

***FINDING THE CENTRE AND ANGLE OF ROTATION***

***Summary:***

***1.*** *When the object and its image are given****,*** *the centre of rotation is located as follows****:***

***(i)*** *Join two corresponding points of the object and the image shape*

***(ii)*** *Construct a perpendicular bisector of this line*

***(iii)*** *Repeat for another pair of corresponding points*

***(iv)****The perpendicular bisectors meet at the centre of rotation*

***2.*** *To find the angle of rotation****,*** *you measure the angle formed by joining corresponding points to the centre of rotation****.*** *This angle may be positive or negative depending on the direction of rotation*

***EXAMPLES:***

***1.*** *A triangle with vertices* ***P(5, 4), Q(8, 4)*** *and* ***R(5, 7)*** *is mapped onto its image*   *and*  *after a rotation****.***

***(i)*** *Draw the two triangles on the same axes****.******[****Use a scale of* ***1cm*** *to* ***1*** *unit****]***

***(ii)*** *Find the centre and angle of rotation*

***2.*** *A line segment with end points* ***P(−6, 2)*** *and* ***Q(−8, 4)*** *is mapped onto its image*   *and*  *after a rotation****.*** *The image of* ***PQ*** *is further rotated through a half turn to give the image* 

***(i)*** *Draw the two lines on the same axes****.******[****Use a scale of* ***1cm*** *to* ***1*** *unit****]***

***(ii)*** *Find the centre and angle of rotation*

***(iii)*** *Find the coordinates of* 

***2.*** *A line segment with end points* ***P(−6, 2)*** *and* ***Q(−8, 4)*** *is mapped onto its image*   *and*  *under a rotation****.*** *The image of* ***PQ*** *further undergoes a clockwise rotation of* ***60°*** *to give the image* 

***(i)*** *Draw the two lines on the same axes****.******[****Use a scale of* ***1cm*** *to* ***1*** *unit****]***

***(ii)*** *Find the centre and angle of rotation*

***(iii)*** *Find the coordinates of*  *State the angle formed between* ***PQ*** *and* 

***EER:***

***1.*** *Find the coordinates of the image of a triangle with vertices* ***P(8, 6), Q(4, 10)*** *and* ***R(2, 6)*** *after a half turn about point* ***(3, 2).***

***2.*** *A triangle with vertices* ***P(1, 1), Q(2, 4)*** *and* ***R(4, 0)*** *undergoes a positive rotation of* ***90°*** *about the origin to give triangle*  *Triangle* *is then reflected in the line* ***y = −x*** *to give triangle* 

***(i)*** *Draw the three triangles on the same axes****.******[****Use a scale of* ***1cm*** *to* ***1*** *unit****]***

***(ii)*** *Use your graph to describe fully a single transformation which is equivalent to the two successive transformations*

***(iii)*** *Find the matrix of a single transformation which maps* ***PQR*** *onto* 

***3.*** *A triangle with vertices* ***P(3, 2), Q(8, 4)*** *and* ***R(5, 7)*** *is mapped onto its image by a positive quarter turn about the origin****.***

***(a)*** *Write down the matrix for the rotation*

***(b)*** *Find the coordinates of the image of* ***PQR:***

***(i)*** *using matrices*

***(ii)*** *by construction*

***4.*** *A triangle with vertices* ***P(2, 3), Q(2, 2)*** *and* ***R(4, 2)*** *is mapped onto its image*   *and*  *under a rotation****.*** *The image of triangle* ***PQR*** *further undergoes a rotation of* ***52°*** *to give the image* 

***(i)*** *Draw the triangle* ***PQR*** *and its image* *on the same axes****,***

***[****Use a scale of* ***2cm*** *to* ***1*** *unit****]***

***(ii)*** *Find the centre and angle of rotation*

***(iii)*** *Find the coordinates of*  *State the angle formed between* ***PQR*** *and* 

***5.******(a)*** *Use the points* ***I(1, 0)*** *and* ***J(0, 1)*** *to find the matrix corresponding to****:***

***(i)*** *a reflection in the line* ***x + y = 0***

***(ii)*** *a positive quarter turn about the origin*

***(b)*** *Find the coordinates of the image when the points****:***

***(i)******P(2, 2)*** *and* ***Q(4, 2)*** *undergo a reflection in the line* ***x + y = 0*** *to give*

*and* 

***(ii)****and* *undergo a positive quarter turn about the origin to give*

*and* 

***(c)*** *By plotting* ***PQ*** *and its images on the same axes****,*** *describe a single transformation that would map* *back onto* ***PQ***

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